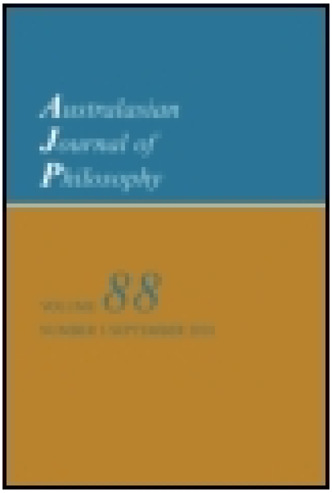

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What Logics Mean: From Proof Theory to Model-Theoretic Semantics, by James W. Garson

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BOOK REVIEW

Garson, James W., *What Logics Mean: From Proof Theory to Model-Theoretic Semantics*, Cambridge: Cambridge University Press, 2013, pp. xv + 285, £10.99 (paperback).

Thirty years ago, most logicians assumed that, while proof theory may give us tools to prove theorems, when we want to deal directly with semantics we must go into model theory. But more recently, partly in connection with the post-Wittgensteinian boom in the popularity of various use theories of meaning, proof theory has started to claim a greater import, and there have appeared concepts such as that of *proof-theoretic semantics*, which would hardly have been conceivable earlier.

But even if semantics may ultimately be fully grounded in rules, it still seems that its model-theoretic presentation is the most convenient. Therefore, the question arises of whether we can transform a proof-theoretical delimitation of meaning into a model-theoretic shape. (Hence, it is worthwhile, as Garson puts it [5] by quoting Sundholm [1986: 478], to find a way to '*read off* a [model-theoretic] semantics from the ... rules'.) In some simple cases, we can answer affirmatively (for example, the standard introduction and elimination rules for conjunction lead us directly to the classical truth table); in other cases, however, the situation is much more complicated.

(Interestingly, while discussing ways of transforming the question *Which kinds of meanings can be conferred on words by means of inferential rules*? into a more rigorously manageable shape, I ended up with a framework almost identical to that employed by Garson [Peregrin 2006]. It should also be noted that the general representation of semantics in terms of sets of admissible truth valuation, which is an integral part of this framework, was put forward by van Fraassen [1971] and then by Dunn and Hardegree [2000]; while the idea of seeing inferential rules as carving spaces of admissible valuations was first systematically analyzed by Scott [1971, 1972].)

In his book, Garson approaches this question on a very general level: he tries to show how some inferential patterns induce a kind of 'natural semantics'. In particular, he shows how sets of inferential rules can be seen as carving out spaces of admissible truth valuations, and how sometimes these spaces can be further transformed into something resembling semantics, in the traditional sense of the word. The book abounds with interesting results, but its main virtue is the systematicity with which it tackles a broad range of problems associated with the relationship between proof theory and model theory.

What Garson calls an *argument* is a step from a list of formulas (premises) to a formula (conclusion): $A_1, \ldots, A_n \vdash A$. (I would prefer to call this, not an *argument*, but rather an *inferential rule*, for the author's terminology seems to me to be potentially misleading. However, I will try to stay with the terminology of the book.) We can say that such an argument is satisfied by a valuation v if it is not the case that $v(A_1) = 1, \ldots, v(A_n) = 1$, and $v(A) = 0$. And we can say that the argument renders inadmissible—and hence excludes—all valuations that do not satisfy it. In this way, we can see any set of arguments as delimiting a certain range of admissible valuations.

Now, the undeniable fact is that no inferential rules enable us to delimit precisely the range of all and only valuations that comply with the classical truth tables. Take two sentences, A and B, and imagine that we want B to be the classical negation of A, i.e. that we want to exclude all valuations that map both A and B onto 0 and also those that map both of them onto 1:

	A	B
~~1.~~	~~1~~	~~1~~
2.	1	0
3.	0	1
~~4.~~	~~0~~	~~0~~

And it can be shown that no set of arguments can do this. (Why? One way to explain this—a way not explicitly invoked in Garson's book—is to use a theorem due to Hardegree [2005], which says that no argument can drive a wedge between a set of valuations and their supervaluation, that is, the valuation that maps an element onto 1 just in the case where all valuations of the set map it onto 1. This implies that, when using arguments, we have no way of excluding the fourth row without excluding also the second or the third; and we have no way of excluding the first one, for it is the supervaluation of the empty set of valuations.)

How can we carve the space of admissible valuations so that it does contain all and only the valuations that comply with the classical truth tables? One easy way is to move from arguments to sequents (and thus from natural deduction to sequent calculus). Then we can have this:

$$A, B \vdash$$
$$\vdash A, B$$

The first of the sequents guarantees that A and B cannot both be 1, while the second guarantees that they cannot both be 0. (A question that may arise in this context is whether sequents allow us to delimit any conceivable set of valuations; and the answer is that this is the case only when we allow for an infinite number of formulas on the left, as well as on the right, of $\vdash$ (see Peregrin [2010]).

Garson does not want to go the way of the sequent calculus and so he explores another route. It is obvious that the arguments delimiting the semantics of a language are usually generated from a finite basis by a set of rules. (If arguments were called, as I would propose, *rules*, then these rules would have to be called *metarules*.) But while we can take the rules as just a way of producing all of the arguments that delimit the semantics (which underlies the definition of what Garson calls *deductive validity*), we can also consider them as taking a direct part in the delimitation. The point is that a rule can be construed as excluding every valuation that satisfies the arguments that are its premises, while it does not satisfy the argument that is its conclusion. (In this way, we reach the concept of what Garson calls *local validity*.) Thus, for example, the rule

$$A \vdash B$$
$$A \vdash \neg B$$
$$\vdash \neg A$$

excludes every valuation v that satisfies $A \vdash B$ and $A \vdash \neg B$ and does not satisfy $\vdash \neg A$. This means that a valuation is admissible iff $v(A) = 1$ and $v(B) = 0$, or

$v(A) = 1$ and $v(\neg B) = 0$, or $v(\neg A) = 1$. And this holds iff no admissible valuation maps both a sentence and its negation onto 0.

How do we exclude a valuation that maps both a sentence and its negation onto 1? Instead of $A, B \vdash$ we can have $A, B \vdash C$, which, however, brings about the needed effect only if we know that every valuation maps at least one sentence onto 0, which Garson achieves stipulatively, simply by not considering the function mapping all sentences onto 1 as a valuation. (It is up to the reader to judge in how far this is a kind of 'cheating'.)

The fact that the rules of natural deduction, construed in this way, pinpoint classical semantics is surprising; but Garson does not overestimate it, for this way of interpreting the rules is in certain respects problematic. What he thinks is a more reasonable construal of the rules is to take them to be satisfied by (and hence to exclude) not individual valuations, but *sets* of valuations. A set of valuations is taken to satisfy a rule iff either not all of them map all the premises of the rule onto 1, or all of them map its conclusion onto 1. A set of valuations then satisfies a rule if it either does not satisfy all the antecedent arguments or satisfies the consequent argument; and it is excluded by the rule iff it does not satisfy it. This is, finally, what Garson calls *global validity*.

This construal of rules appears to be more adequate, but unfortunately it again does not allow us to pinpoint precisely the classical valuations. Of course, rather than seeing this as a shortcoming, we can draw the moral that the natural semantics for the logic based on arguments (single-conclusion inferential rules) is indeed the intuitionist one. But this conclusion is somewhat abstract; intuitionist logic does not have a semantics similar to the classical logic, such that we would be able to say that the valuations that are not excluded by the inferential rules are precisely 'the intuitionist ones'.

Garson, however, gives it a much more concrete shape: namely, he shows a sense in which it is the Kripkean semantics for intuitionist logic that is yielded directly by the usual delimitation of logic by the rules of natural deduction. In particular, he considers the relation $\leq$ between valuations such that $v \leq v'$ iff $v(A) = 1$ implies $v'(A) = 1$ for every A, and he shows that, for a set of rules satisfying the rules of natural deduction, it is the case that $v(\neg A) = 1$ iff $v'(A) = 0$ for every v' such that $v \leq v'$. (What is slightly surprising—at least to me—is that the inferential rules Garson employs are infinitistic, i.e. he does not assume that the antecedents of the rules are finite. It is surprising, especially, for I do not why he needs this—it would seem to me that what he proves would hold even if the rules were finitistic.)

Garson's book contains a lot of other interesting material. He applies his methods not only to classical and intuitionist logic, but also, to some extent, to modal logic, to predicate logic, and to some less usual systems, such as the logic of vagueness and his own logic of 'open futures'. On the whole, his book presents an admirable self-contained theory. Thus, he succeeds in showing us, in detail, how to '*read off* a [model-theoretic] semantics from the . . . rules'; and this, in my opinion, is certainly no minor achievement.

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